## Total variation of the Hessian determinant

Assume that  $\Omega$  is an open subset of  $\mathbb{R}^n$  and  $u: \Omega \to \mathbb{R}$  is  $C^2$  function. Writing  $D^2u$  to denote the matrix of second partial derivatives of u, a version of the Change of Variables formula for multiple integrals implies that

(1) 
$$\int_{x\in\Omega} |\det D^2 u| \ dx = \int_{p\in\mathbb{R}^n} \#\{x\in\Omega: Du(x)=p\} \ dp,$$

where for any set S,

$$\#S := \begin{cases} \text{the cardinality of } S & \text{if } S \text{ is finite} \\ +\infty & \text{otherwise.} \end{cases}$$

Equation (1) contains the assertion that the integrand on the right-hand side is integrable.

A long-term goal is to make sense of the right-hand side of (1) for the largest natural space of functions, and to investigate properties of functions for which this integral is finite. This project will focus on this goal in the case when the function u is piecewise linear, and will address the following concrete issues:

- (1) assigning a meaning to the integral  $\int_{p\in\mathbb{R}^n}\#\{x\in\Omega:Du(x)=p\}\,dp$  when u is piecewise linear, and
- (2) developing and implementing numerical methods for computing  $\int_{p \in \mathbb{R}^n} \#\{x \in \Omega : Du(x) = p\} dp$  when u is piecewise linear, starting in n = 2 dimensions.

An abstract answer to the first question is known, so the challenge is to refashion it into something more workable.

Work on this project will require some knowledge of real analysis, some programming experience, and a willingness to learn.

This research is connected to another project for this summer, **Refinements** of Alexandrov's inequality, and students working on the two projects should expect to talk frequently and exchange ideas.

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