



## Departmental PhD Thesis Exam

Tuesday, April 14th, 2026 at 10:00 a.m.  
(sharp) via Zoom / BA6180

PhD Candidate : George Huang

Supervisor : Professor Henry Kim

Thesis title : Scattering Constants and Cuspidal Divisor Class Groups of Some Noncongruence Subgroups



### Abstract

This thesis presents computations involving certain noncongruence subgroups of the modular group. Using the Kronecker limit formula, we calculate the scattering constants for three families of nonelliptic noncongruence subgroups. For these same groups, we also determine their cuspidal divisor class groups and show that these groups are finite.

These findings relate to the Manin–Drinfeld theorem, which states that cuspidal divisor class groups are finite for congruence subgroups. Since no general finiteness result is known for noncongruence subgroups, these examples provide additional insight into their arithmetic properties.

Furthermore, we compute cuspidal divisor class groups for several congruence and noncongruence subgroups associated with elliptic curves. In some of the noncongruence cases, the cuspidal divisor class groups are finite, indicating that finiteness can occur outside the congruence setting.

Overall, these computations offer explicit data in a field where such examples are relatively rare and may contribute to further understanding of noncongruence modular curves.