

RIGIDITY AND DETERMINATION FOR BILLIARDS

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ABSTRACT. The purpose of this project is to work towards a positive answer to conjectures about length rigidity and determination for mathematical convex billiards. The techniques that will be developed have to do with Functional Analysis, but they will mostly be developed from scratch.

1. DESCRIPTION

There is a very popular (and influential) question which was posed in the late 60's by Mark Kac: *Can one hear the shape of a drum?*. This is an example of an *inverse problem*: one has an object (in this case a planar domain in $\mathbb{R}^2$), extracts some seemingly coarse information (in this case the set of eigenvalues of the Laplacian operator with given boundary conditions), and tries to understand to which extent it is possible to reconstruct the original object from this data. While the answer to above question is still largely unknown (if framed sensibly), there are related questions that one can try to understand.

Among such questions, there is the problem of dynamical determination (or length-spectrum determination) of manifold or billiards. In this project we aim to further develop a modern approach to the problem of billiard determination; there are several direction in which progress can be made: from numerical simulations to full-fledged arguments. Most of the recent progress has been made using some functional analytic techniques (Banach spaces and operators), which are fairly elementary and can be easily understood and put to further use.

We will work on improving such techniques, using a mix of Real Analysis tools, creativity and possibly inspiration from numerical simulations. The outcome of a related project was fairly recently published in a peer-reviewed journal: that is to say that there is lots of interest in the subject and lots of ongoing activity!