DEPARTMENT OF MATHEMATICS
Real Analysis Comprehensive Exam 2025
September 15, 2025 - 6:00 - 9:00 p.m.
BA6183

NO AIDS ALLOWED. Passing score is 80 percent.

Last name ....

First name ....

Email ....

1. Suppose $E \subset \mathbb{R}$ has positive Lebesgue measure, and A is a countable dense subset of $\mathbb{R}$. Show that $m(\mathbb{R} \setminus \bigcup_{a \in A}(a + E)) = 0$.

2. Let $f \in L^1(\mathbb{R})$. Show that there is a subsequence $n_k \to \infty$ such that $f(\cdot + 1/n_k) \to f$ a.e.

3. Suppose $f : [1, \infty) \to \mathbb{R}$ be a C^1 function satisfying $f(1) = 0$ and $f' \in L^2$. Show that $f(x)/x \in L^2([1, \infty))$.

4. State the open mapping theorem and the closed graph theorem. You do not need to prove it.

5. Define the Fourier transform of a function $f \in L^1(\mathbb{R}^n)$ and show that it is a continuous linear mapping from $L^1(\mathbb{R}^n) \to L^\infty(\mathbb{R}^n)$. Is this mapping injective or surjective? Why?

6. Do the following functions define tempered distributions? Prove or disprove your claim.

   - $f(x) = x^2 \sin(x)$.
   - $f(x) = e^x$.