UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Monday, February 12th, 2024 at 10:00 a.m. (sharp)
via Zoom / BA6183

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Supervisor : Dmitry Panchenko

Thesis title : A Hamilton-Jacobi approach to the stochastic block model

Abstract

This thesis addresses the problem of recovering the community structure in the stochastic block model with two communities. The stochastic block model is a random graph model with planted clusters widely employed as the canonical model to study clustering and community detection. The focus is on the fundamental limits of community detection, quantified by the asymptotic mutual information between the observed network and the actual community structure. This mutual information is studied using the Hamilton-Jacobi approach, pioneered by Jean-Christophe Mourrat.

The first contribution of this thesis is a detailed description of the Hamilton-Jacobi approach, and its application to computing the limit of the mutual information in the dense stochastic block model, where the average degree of a node diverges with the total number of nodes. The main novelty is a well-posedness theory for Hamilton-Jacobi equations on positive half-space that leverages the monotonicity of the non-linearity to circumvent the imposition of an artificial boundary condition as previously done in the literature.

The second contribution of this thesis is a novel well-posedness theory for an infinite-dimensional Hamilton-Jacobi equation posed on the set of non-negative measures and with a monotonic non-linearity. Such an infinite-dimensional Hamilton-Jacobi equation appears naturally when applying the Hamilton-Jacobi approach to the sparse stochastic block model, where the total number of nodes diverges while the average degree of a node remains bounded. The solution to the infinite-dimensional Hamilton-Jacobi equation is defined as the limit of the solutions to an approximating family of finite-dimensional Hamilton-Jacobi equations on positive half-space. In the special setting of a convex non-linearity, a Hopf-Lax variational representation of the solution is also established.

The third contribution of this thesis is a conjecture for the limit of the mutual information in the sparse stochastic block model, and a proof that this conjectured limit provides a lower bound for the asymptotic mutual information. In the case when links across communities are more likely than links within communities, the asymptotic mutual information is known to be given by a variational formula. It is also shown that the conjectured limit coincides with this formula in this case.