UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Friday, May 23rd, 2025 at 10:00 a.m. (sharp)
via Zoom / BA6183

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Supervisor : Yevgeny Liokumovich

Thesis title : On Density and Equidistribution of Stationary Geodesic Nets

Abstract

Stationary geodesic nets are embedded graphs in a Riemannian manifold (M^n, g) which are stationary with respect to the length functional. In this thesis, we study the distribution of closed geodesics and stationary geodesic nets in Riemannian manifolds. We prove that for a generic set of metrics on a closed manifold M^n, $n \geq 2$, the union of all the embedded stationary geodesic nets in (M^n, g) forms a dense subset of M^n. For $n = 2$, we prove that for generic metrics on M^2 we can obtain an equidistributed sequence of closed geodesics. This means that there exists a sequence of closed geodesics $\{\gamma_i\}_{i\in\mathbb{N}}$ such that for every open subset U of M^2,

$$\lim_{k\to\infty} \frac{\sum_{i=1}^{k} \mathrm{L}_g(\gamma_i \cap U)}{\sum_{i=1}^{k} \mathrm{L}_g(\gamma_i)} = \frac{\mathrm{Vol}_g(U)}{\mathrm{Vol}_g(M)}.$$

We show that the previous equidistribution result also holds for $n \geq 3$ but replacing closed geodesics by stationary geodesic nets. The main tool that we use is Almgren-Pitts Min-Max Theory, in particular the Weyl law for the volume spectrum. We also prove a Structure Theorem for stationary geodesic nets analogous to that of Brian White for minimal submanifolds, which is used to prove the density and equidistribution results. The density result was obtained in collaboration with Yevgeny Liokumovich, and the equidistribution result in dimensions 2 and 3 is joint work with Xinze Li.