



Departmental PhD Thesis Exam

Monday, March 30th, 2026 at 2:00 p.m. (sharp)
via Zoom / BA6183

PhD Candidate : Hymn Chan

Supervisor : Florian Herzig

Thesis title : On Breuil's Lattice Conjecture for GL_2



Abstract

Roughly speaking, the p -adic Langlands Program seeks a correspondence between n -dimensional Galois representations $\rho : G_K \rightarrow GL_n(\overline{\mathbb{Q}}_p)$ and certain admissible unitary Banach space representations of $GL_n(K)$ over some p -adic field, where K/\mathbb{Q}_p is an unramified extension. Let $r : G_F \rightarrow GL_2(\overline{\mathbb{Q}}_p)$ be a global Galois representation associated to an automorphic form π , where p is inert in F . One can construct a unitary Banach space representation $\pi(r)$ of $GL_2(F_p)$ associated to r . The p -adic Langlands Program predicts that $\pi(r)$ depends only on r restricted to p .

This is currently beyond our reach, but one can instead study the locally algebraic type, a $GL_2(\mathcal{O}_K)$ representation inside $\pi(r)$. Motivated by local-global compatibility, Breuil conjectured that the $GL_2(\mathcal{O}_K)$ -invariant integral lattice inside the locally algebraic type induced by the completed cohomology with integral coefficients is uniquely determined by r restricted to p . This conjecture provides evidence for the p -adic Langlands correspondence and the local-global compatibility. In this thesis, we prove Breuil's Lattice Conjecture for higher Hodge–Tate weights, which are small relative to p . We further prove that the patched modules of all lattices inside these locally algebraic types with irreducible cosocle are cyclic.

One key input of the proof is a structure theorem for mod p representations of $GL_2(\mathcal{O}_K)$, which are residually multiplicity free and of finite length. Another input is an explicit computation of universal framed Galois deformation rings, which parameterize potentially crystalline lifts with fixed tame inertial types and higher Hodge–Tate weights.