



Departmental PhD Thesis Exam

Monday, August 26th, 2024 at 2:00 p.m. (sharp)
via Zoom / BA6183

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Supervisor : Catherine Sulem

Thesis title : Nonlinear Modulation of Surface Water Waves over a Periodic Bottom



Abstract

This thesis contributes to the study of the two-dimensional water wave problem in the presence of a variable bottom topography, which describes the motion of a free surface over a body of water under the influence of gravity. When the bottom topography is flat (with depth possibly infinite), it is known that, in a weakly nonlinear regime, the envelope of modulated surface waves is governed by the cubic nonlinear Schrödinger (NLS) equation. This result was derived in the case of infinity depth by Zakharov (1986), and later extended to finite depth domains over a flat bottom by Hasimoto and Ono (1978).

In this thesis, we extend this derivation to the two-dimensional water wave problem in the case of a variable bottom, assumed to be a smooth periodic function. Starting from the Zakharov/Craig-Sulem formulation of water waves, we use a multiple-scale method to write the surface wave in the form of a slowly modulated Bloch-Floquet wavepacket, which propagates at the group velocity. We show that the envelope of wave amplitude is governed by the NLS equation. A key step in this process is to investigate the actions of the Dirichlet-Neumann operator on multiple-scale functions of various forms.

We also present perturbative calculations of the Bloch-Floquet eigenvalues and eigenfunctions of the Dirichlet-Neumann operator when the variation of periodic bottom is small. These are used to study the effect of the variable bottom on the coefficients in the NLS equation.