



## Departmental PhD Thesis Exam

Wednesday, August 28th, 2024 at 11:00 a.m. (sharp)  
via Zoom / BA6183

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Supervisor : Eckhard Meinrenken

Thesis title : On Multiplicative Weightings  
for Lie Groupoids and Lie Algebroids



### Abstract

We present a thorough study of the differential geometry of weightings and develop the theory of weightings for vector bundles, Lie groupoids, and Lie algebroids.

We begin by extending the work of Loizides and Meinrenken on weighted manifolds. We define weighted submanifolds, weighted immersions, and weighted embeddings, and prove normal form theorems for these objects. We also study characterizations of weighted morphisms in terms of their graphs and in terms of weighted paths. We further extend the theory of weighted manifolds by developing a theory of linear weightings for vector bundles.

Following this, we give three equivalent definitions of a multiplicative weighting for a Lie groupoid  $G \rightrightarrows M$ : one involving the structure maps for the Lie groupoid, one involving the graph of the groupoid multiplication, and one involving the weighted deformation space. We also include a discussion of weighted VB-groupoids, and prove some basic theorems involving these objects.

In the last two chapters of this thesis, we study infinitesimally multiplicative weightings for Lie algebroids. We characterize these in terms of linear Poisson structures and homological vector fields. We show that multiplicative weightings differentiate to infinitesimally multiplicative weightings and solve the integration problem for infinitesimally multiplicative weightings along wide Lie subalgebroids. In particular, we classify multiplicative weightings of a Lie groupoid along its units in terms of Lie filtrations of its Lie algebroid. We make progress towards a solution for the general integration problem by giving a condition for when an infinitesimally multiplicative weighting along a general Lie subalgebroid integrates.