## USRA project "Groupoids in Mathematical Physics" (with Boris Khesin)

## Project description:

The project is aimed at explicit constructions of groupoids and algebroids that serve as symmetry frameworks for various equations of mathematical physics. It will particularly focus on extending Arnold's geodesic approach to the Euler equation in fluid dynamics with fixed boundaries. Until recently that approach has been limited to systems whose symmetries form a group. On the other hand, for instance, the motion of a fluid with free moving boundary is described by a trajectory on a groupoid, as only some of the flow maps admit composition.

One of the main directions of the summer project is studying the groupoidalgebroid literature and extending the groupoid program proposed in [1,2] for generalized flows and vortex sheets in several directions. Here are several key questions: How to include "broken Virasoro" groupoid and singular solution of the KdV equation, how to include moving boundary and brackets for water waves? How to include surface tension? Could one apply the groupoid approach to study the Kelvin-Helmholtz instabilities of vortex sheets, maybe in one-two dimensional settings?

## **References:**

Anton Izosimov and Boris Khesin: Vortex sheets and diffeomorphism groupoids, Advances in Math., vol.338 (2018), 447-501; arXiv:1705.01603

Anton Izosimov and Boris Khesin: Geometry of generalized fluid flows, Calc. Var. Partial Differential Equations, vol.63:3 (2023), 30pp.; arXiv:2206.01434