UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Tuesday, September 9, 2025 at 10:00 a.m. (sharp)
via Zoom / BA6180

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Supervisor : Almut Burchard

Thesis title : Dense Forests with Low Visibility

Abstract

A dense forest in $\mathbb{R}^d$ is a set of points with finite density such that any line segment of length $V(\varepsilon)$, the visibility function of the forest, comes within distance ε of an element of the set—the points with a ball of radius ε centred at them can be viewed as trees obstructing visibility, hence the name. Such sets are known to exist. The problem now is to determine how small the visibility can be made and how many additional properties can be required from a dense forest. This thesis presents results in those directions.

In this thesis, a dense forest is constructed by modifying a base set obtained from a Poisson point process by adding some points to it such that the visibility is in $O\left(\varepsilon^{-(d-1)} \ln \varepsilon^{-1}\right)$. Further, the Poisson point process can be adjusted so that the base set is uniformly discrete while maintaining the same final visibility. Due to the latter addition of points, the resulting dense forest is not uniformly discrete.

Through a slight change of perspective—from eliminating lines of sight to eliminating empty regions—the problem of finding dense forests is closely related to Danzer's problem. It asks if there exists a set of finite density intersecting every convex set of volume 1. A relaxation of Danzer's problem is examined where the volume of the convex sets is allowed to grow as their eccentricity increases. Bounds on the growth of the volume are obtained by adapting the approach taken for dense forests earlier.