NSERC USRA 2025 Proposal
Computations with L-functions

Knowledge about the statistics of zeros of L-functions, like the Riemann zeta function, inform our understanding on the equidistribution of primes in many contexts. There are certain notoriously troublesome zeros, known as Landau-Siegel zeros, which pose a significant potential barrier to this equidistribution. Landau-Siegel zeros conjecturally do not exist, but this problem has remained open for 100+ years. It is also surprisingly connected to many other central problems in number theory, such as the twin prime conjecture.

Computationally speaking, the generalized Riemann hypothesis (GRH) has been verified up to a certain height for finitely many L-functions by several authors, notably Platt (2013). GRH up to a fixed height implies Landau-Siegel zeros do not exist, so the conjecture has been verified for these L-functions.

This project aims to computationally prove that Landau-Siegel zeros do not exist for more L-functions than were previously known by applying new methods to the subject. There will be several key components, such as establishing key explicit analytic estimates, implementing fast computational algorithms to verify the non-existence, and exploring relationships with class groups. We will collaboratively decide on your research path. A solid background with complex variables and elementary number theory is strongly recommended.

First, we will establish foundational knowledge for Landau-Siegel zeros and explicit analytic estimates of L-functions. Second, we will study recent techniques for your proposed investigations and apply them to our research problem. Third, we will learn how to effectively program for high precision numerical calculations and parallel computing on large clusters. Fourth, you will summarize the outcomes of your research in an appropriate academic forum, so the community can use this work to guide future research.

If you are interested, please submit this form:

https://forms.office.com/r/Z4gfeRfPmY

If you have any further questions, please feel free to contact me.

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