DEPARTMENT OF MATHEMATICS
University of Toronto

Complex Analysis Exam

1.5 hours

September 3, 2014

There are three questions, all of equal value.
Show all your work.

1. Evaluate via residues
$$\int_0^\infty \frac{x^{a-1}}{1+x}dx$$
where $0 < a < 1$.

2. Suppose that Ω is a domain in $\mathbb{C}$, f_k is a sequence of analytic functions on Ω, $f_k \to f$ uniformly on compact subsets of Ω, and f has a zero of order N at $z_0 \in \Omega$. Show that there exists $\rho > 0$ such that for k sufficiently large, f_k has exactly N zeros counting multiplicities on $|z - z_0| < \rho$.

3. a) Let f and g be $1-1$ analytic mappings from a domain $\Omega \subset \mathbb{C}$ onto the unit open disc $\Delta \subset \mathbb{C}$. Suppose that for some point $z_0 \in \Omega$, $f(z_0) = g(z_0) = 0$. What is the relation between f and g?

   b) Let f be a $1-1$ analytic map of the unit disc Δ onto the unit square with centre 0, satisfying $f(0) = 0$. Show that $f(iz) = if(z)$.