1. Evaluate
\[ \int_0^{2\pi} \frac{\cos \theta}{3 + \cos \theta} d\theta. \]

2. Show that there is a complex analytic function defined on the set
\[ U = \{ z \in \mathbb{C} : |z| > 4 \} \]
whose derivative is
\[ \frac{z}{(z-1)(z-2)(z-3)}. \]

Is there a complex analytic function on \( U \) whose derivative is
\[ \frac{z^2}{(z-1)(z-2)(z-3)}? \]
Explain your answer.

3. Let \( S \) be the half-strip \( S = \{ z = x + iy : |x| < 1, y > 0 \} \) and let \( f \) be an
analytic function defined on \( S \) such that
a) \( |f(z)| \leq 2, z \in S; \)
b) \( \lim_{y \to \infty} f(iy) = 1. \)
Prove that for any \( 0 < a < 1, \lim_{y \to \infty} f(x + iy) = 1 \) uniformly for \( |x| \leq a. \)
(\textit{Hint: consider the family of functions } f_t : S \to \mathbb{C}, f_t(z) := f(z + it), t \geq 0.)

4. Let \( f : \mathbb{C} \to \mathbb{C} \) be an entire function. Prove that if \( |f(z^2)| \leq 2|f(z)| \) for all
\( z \in \mathbb{C}, \) then \( f \) is constant.