UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Friday, September 12, 2025 at 10:00 am (sharp)
via Zoom / BA6183

PhD Candidate : Reila Zheng

Supervisor : Giulio Tiozzo

Thesis title : Combinatorial Structure of Parameter Spaces of Polynomials

Abstract

In this thesis, we will explore the combinatorics of the Mandelbrot set. Firstly, I will discuss relates the ordering of the hyperbolic components along veins to a classical result of Sharkovsky of orbit forcing for maps of one real dimension. In this problem I will assume V is a principal vein of combinatorial rotation number p/k (and denote $i = 1$), or an i-th secondary vein for $2 \leq i \leq k-1$. Suppose $C_V(n)$ is the hyperbolic component of period n along V closest to the main cardioid. I will show that $C_V(n) \succ_V C_V(m)$ if $n >_k m$. Additionally, the hyperbolic components of $C_V(n)$ with $n \equiv i \mod k$ are minimal and form a dynamically simple family whose Hubbard trees are spiral graphs.

Secondly, I will discuss a characterization of a family of abstract Hubbard trees by their topology. For any $n \geq 3$, let $\mathscr{M}(n)$ denote the set of parameters c in the Mandelbrot set with Hubbard tree T_c homeomorphic to an n-star. An n-vein is a union of veins in the Mandelbrot set that is homeomorphic to an n-star. I will show that the set of parameters of $\mathscr{M}(n)$ is a exactly the union of $\phi(n)$ n-stars and the hyperbolic components that pass through them. Additionally, I will identify a family of dynamically simple Misiurewicz tips that define the n-veins.