Abstract

In this thesis, we classify generic coadjoint orbits for the action of symplectic (equivalently, area-preserving) diffeomorphisms of compact symplectic surfaces with or without boundary. This completes V. Arnold’s program of studying Casimir invariants of incompressible fluids in 2D. To obtain this classification, we first solve an auxiliary problem, which is of interest by itself: classify generic Morse functions on surfaces with respect to the action of area-preserving diffeomorphisms. As a technical tool, we prove an analog of Morse-Darboux lemma in the case of a singular point on the boundary. We also generalize all the results above to the case of non-orientable surfaces without boundary.