UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE | Department of Mathematics

Departmental PhD Thesis Exam

Tuesday, August 19th, 2025 at 2:00 p.m. (sharp)
via Zoom / BA6183

PhD Candidate : Iván Camilo Salgado

Supervisor : Robert Jerrard

Thesis title : Approximate Solutions to the Superconducting Interface Model

Abstract

We examine various aspects of the *Superconducting Interface Model*

$$\partial_t^2\varphi - \Delta_x\varphi + \frac{1}{\varepsilon^2}\left[\lambda_\varphi(\varphi^2-1)+\beta|\sigma|^2\right]\varphi = 0, \qquad \text{in } (0,T)\times\mathbb{R}^n,$$
$$\partial_t^2\sigma - \Delta_x\sigma + \frac{1}{\varepsilon^2}\left[\lambda_\sigma(|\sigma|^2-m_\sigma^2)+\beta\varphi^2\right]\sigma = 0, \qquad \text{in } (0,T)\times\mathbb{R}^n,$$

for $0<\varepsilon\ll 1$, for certain choices of positive parameters $(\lambda_\varphi,\lambda_\sigma,m_\sigma,\beta)$, and with (φ,σ) taking values in $\mathbb{R}\times\mathbb{C}$. This model was introduced by Kyle Thompson in his doctoral thesis, where he proved, modulo a spectral assumption, the existence of equivariant solutions for the particular case $n=2$ such that $\varphi\approx+1$ or $\varphi\approx-1$ except in a transition layer of thickness $O(\varepsilon)$ around a hypersurface Γ, and σ is exponentially small except near Γ. In this context, σ is interpreted as carrying a superconducting current confined to the transition layer around Γ. A main contribution of Thompson's original work was to derive the laws of motion, in the form of an ODE, according to which the geometry of Γ changes due to the flow of current within Γ, in the limit $\varepsilon\to 0^+$.

In this thesis, we extend these results in several ways. First, we carry out a careful analysis of the static 1-dimensional superconducting interface model, including a proof of the spectral condition assumed in Thompson's thesis for a range of the model parameters. This completes the rigorous derivation of the laws of motion in the 2-dimensional equivariant case. Additionally, in arbitrary dimensions and without any symmetry assumptions, we provide a rigorous construction of approximate solutions of arbitrary order for the superconducting interface model, yielding a formal derivation of the laws of motion for a superconducting interface in full generality. Finally, we prove the well-posedness for smooth data of these laws of motion for a suitable choice of gauge.