DEPARTMENT OF MATHEMATICS
Algebra Comprehensive Exam 2025
September 16, 2025 - 6:00 - 9:00 p.m.
BA6183

NO AIDS ALLOWED. Passing score is 80 percent.

Last name .....

First name .....

Email .....

1. Suppose that G is a nonabelian simple finite group.

   a) Show that G cannot have a subgroup of index 2 or 3.

   b) Find an example of a nonabelian simple finite group G and a subgroup $H \leq G$ of index 5.

2. Show that the ring $\mathbb{Q}[x]/(x^2-1)$ has precisely four idempotents and that the ring $\mathbb{Z}[x]/(x^2-1)$ has precisely two idempotents. (Recall that an *idempotent* of a ring R is an element $a \in R$ such that $a^2 = a$.)

3. Suppose that A is an $n \times n$ matrix over $\mathbb{Q}$ such that $A^5 = 2I$ (where I is the identity matrix). Show that n is divisible by 5. Also, write down an example of such a matrix A when $n = 5$.

4. Let L/K be a Galois extension of fields of degree 4. Let $n_{L/K}$ be the number of intermediate subfields of L/K, that is, fields F contained in L and containing K. What are the possible values of $n_{L/K}$? Prove your answer is correct.

5. a) Let R be a Noetherian ring and M a finitely-generated R-module. Show that M is isomorphic to the cokernel of a map of finitely-generated free R-modules $f : N_1 \to N_2$.

   b) Show that if R is a PID one may take f as above to be injective.

6. Write down (with proof) the character table of the symmetric group S_4.