A Variational Problem

Assume that Ω is a bounded, open subset of $\mathbb{R}^n$ and that points $x_1, \ldots, x_N \in \Omega$ and values $y_1, \ldots, y_N \in \mathbb{R}$ are given. For $u \in C^2(\Omega)$ define

$$I[u] := \sum_{i-1}^{N} |u(x_i) - y_i| + \lambda \int_\Omega |\det D^2 u| dx$$

where $D^2 u$ is the Hessian, *i.e.* the matrix of second derivatives, and λ is a parameter.

This project will study mathematical issues related to the question:

(1) *Is* $\inf_{u \in C_0^2(\Omega)} I[\cdot]$ *attained?*

where $C_0^2(\Omega)$ is the space of twice continuously differentiable functions in Ω that vanish on $\partial\Omega$.

The answer to this question is in general "no!", because (possibly among other obstacles) $C_0^2(\Omega)$ is not a good space in which to look for a minimizer. So a first problem is to find a better function space for this minimization problem. This will be bigger space that contains functions that are not C^2. A second problem is to understand what $\int |\det D^2 u|$ means for functions in this bigger space. Progress on these questions has been made in recent undergrad summer research projects, and this new project will either pick up where the earlier ones left off or strike out in new but related directions.

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